MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

DEM5018 – ENGINEERING MATHEMATICS 1

(E18)

6 MARCH 2018
2.30PM – 4.30PM
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages (2 pages with 4 questions and 2 pages for appendix).
2. Attempt **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. Find an equation of the line containing the point (-1,-3) that is perpendicular to the line $2x+y=-2$. Then, sketch the two lines on the same graph. (10 marks)

b. Find the equation of the circle in standard form with the center at $(1,-4)$ that passes through $(-2,-1)$. Then, sketch the circle accordingly. (6 marks)

c. Given $\sin A=\frac{2}{3}$, $\frac{\pi}{2}<A<\pi$; and $\cos B=\frac{4}{5}$, $0<\mathrm{B}<\frac{\pi}{2}$. Find $\sin(A+B)$. (5 marks)

d. Solve the equation $1+4\cos x+1=0$ on the interval $0°\le x\le 360°$. (4 marks)

[TOTAL 25 MARKS]

QUESTION 2

a. Given $z=\frac{3+4i}{1-3i}$ and $w=\frac{2+3i}{2-i}$. Find:

i. z and w in standard form and polar form. (6 marks)

ii. $\overline{zw}$ in standard form. (4 marks)

b. Solve the system of equations given by using **Cramer's Rule**. (15 marks)

$$x-y+2z=3$$
$$2x+3y+z=9$$
$$-x-y+3z=11$$

[TOTAL 25 MARKS]

Continued ...

QUESTION 3

a. Find the value of $g^{-1}(x)$ if given that $g(x)=\sqrt[3]{\frac{9-x^2}{5}}$. Then show that $g^{-1}(-2)=7$. (6 marks)

b. Given that $m(x) = \frac{x^2}{2} - 5x + 1$ and $n(x) = 8x + 17$, find $(n \circ m)(x)$. (4 marks)

c. Given a quadratic function $f(x)=x^2+7x+10$.

i. Determine whether the parabola opens upward or downward. (1 mark)

ii. Find the vertex of the parabola. (4 marks)

iii. Find the x-intercept(s). (3 marks)

iv. Find the y-intercept. (2 marks)

v. Sketch the parabola. (5 marks)

[TOTAL 25 MARKS]

QUESTION 4

a. Differentiate the function $y = \frac{1}{1-\sin x}$ with respect to x. (5 marks)

b. Differentiate $3x^2 - xy + 3y = 7$ by using implicit differentiation. (7 marks)

c. A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by $y = \frac{1}{4}t^4 - 26t^2 + 96t$.

i. Find $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$. (5 marks)

ii. Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. (6 marks)

iii. Find the rate of change of y with respect to t when $t = 1$. (2 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDIX

Standard Form of Circle Equation, $(x-h)^2+(y-k)^2=r^2$

Fundamental Identities for Trigonometric Functions

***Even & Odd Properties*:**

$\sin(-\theta)=-\sin\theta$, $\csc(-\theta)=-\csc\theta$, $\cos(-\theta)=\cos\theta$,
$\sec(-\theta)=\sec\theta$, $\tan(-\theta)=-\tan\theta$, $\cot(-\theta)=-\cot\theta$

Reciprocal identities:

$$\csc\theta=\frac{1}{\sin\theta},\qquad \sec\theta=\frac{1}{\cos\theta},\qquad \cot\theta=\frac{1}{\tan\theta}$$

Pythagorean identities:

$$\sin^2\theta+\cos^2\theta=1,\qquad \tan^2\theta+1=\sec^2\theta,\qquad 1+\cot^2\theta=\csc^2\theta$$

Sum/Difference Formula for Cosine: $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

Sum/Difference Formula for Sine: $\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$

Sum/Difference Formula for Tangent: $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

Double Angle Formula:

$$\begin{aligned}\cos 2A&=\cos^2 A-\sin^2 A\\&=1-2\sin^2 A\\&=2\cos^2 A-1\end{aligned}\quad,\quad \sin 2A=2\sin A\cos A\quad,\quad \tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Half Angle Formula:

$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{2}}\ ,\quad \cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos\alpha}{2}}\ ,\quad \tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Complex Numbers

Relations between Polar and Rectangular Coordinates

$$x=r\cos\theta;\quad y=r\sin\theta;\quad r=\sqrt{x^2+y^2};\qquad \tan\theta=\frac{y}{x}$$

Products and Quotients in Polar Form

$$z_1z_2=r_1r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)];\quad z_2\neq 0$$

DeMoivre's Theorem

Powers of Complex Numbers on Polar Form: $z^n=r^n(\cos n\theta+i\sin n\theta)$

Roots of Complex Numbers in Polar Form:

$$z_k=\sqrt[n]{r}\left[\cos\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)+i\sin\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)\right]\quad\text{where}\quad k=0,\,1,\,2,\,\ldots,\,n-1$$

Differentiation Rules

Derivatives: $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Inverse Trigonometry Function

1. $\dfrac{d}{dx}[\sin^{-1}x]=\dfrac{1}{\sqrt{1-x^2}}$
2. $\dfrac{d}{dx}[\cos^{-1}x]=-\dfrac{1}{\sqrt{1-x^2}}$
3. $\dfrac{d}{dx}[\tan^{-1}x]=\dfrac{1}{1+x^2}$
4. $\dfrac{d}{dx}[\csc^{-1}x]=-\dfrac{1}{x\sqrt{x^2-1}}$
5. $\dfrac{d}{dx}[\sec^{-1}x]=\dfrac{1}{x\sqrt{x^2-1}}$
6. $\dfrac{d}{dx}[\cot^{-1}x]=-\dfrac{1}{1+x^2}$

Hyperbolic Function

1. $\dfrac{d}{dx}[\sinh x]=\cosh x$
2. $\dfrac{d}{dx}[\cosh x]=\sinh x$
3. $\dfrac{d}{dx}[\tanh x]=\sec h^2 x$
4. $\dfrac{d}{dx}[\sec hx]=-\sec hx\tanh x$
5. $\dfrac{d}{dx}[\coth x]=-\csc h^2 x$
6. $\dfrac{d}{dx}[\csc hx]=-\csc hx\coth x$

The Second Derivative Test

Suppose f'' is continuous near c.

If $f'(c)=0$ and $f''(c)>0$, then f has local minimum at c.

If $f'(c)=0$ and $f''(c)<0$, then f has local maximum at c.